

What Quantity is Measured in an Excess Noise Experiment?

U. Gavish, Y. Imry, Y. Levinson
Weizmann Institute of Science
76100 Rehovot, Israel

B. Yurke
Bell Labs, 1C363
Murray Hill, NJ 07974

Consider a measurement in which the current coming out of a mesoscopic sample is filtered around a given frequency, amplified, measured and squared. Then this process is repeated many times and the results are averaged. Often, two such measurements are performed on the same system in and out of equilibrium (the nonequilibrium state can be obtained by a variety of methods, e.g., by applying a DC voltage or electromagnetic radiation to the sample). The excess noise is defined as the difference in the noise between these two measurements. We find that this excess noise is given by the excess of the non-symmetrized power-spectrum of the current-noise. This result holds for a rather general class of experimental setups.

I. INTRODUCTION

Consider a measurement in which the current coming out of a mesoscopic sample is filtered around a frequency $\Omega > 0$, then amplified, measured, squared, then this process is repeated many times and the results are averaged. The final result of such a procedure is called the *current fluctuations*, the *power spectrum*, or the *current noise*, at frequency Ω .

Often, two such measurements are performed on the same system: the first while it is driven out of equilibrium (e.g., by applying a DC voltage¹ or electromagnetic radiation^{2,3} to it) and the second in equilibrium (the voltage source is turned off, i.e., the power supply becomes a short). The *excess noise* is defined as the difference in the noise between the first and the second measurement. The present work analyzes what quantity one should calculate in order to predict excess noise.

In the rest of the introduction some basic concepts in amplification theory are introduced. In Sec. II the measurement procedure is defined. In Sec. III we analyze the classical case and in sections IV and V the quantum one - when $\hbar\Omega$ is comparable with or larger than the temperature, the voltage, or the RF radiation frequency applied to the sample. Finally, a possible verification of the results is presented.

Our main result can be stated as follows. The result of a noise measurement depends on the particular instrumentation used in the setup - what type of amplifier and detector are used, what are their temperatures, etc. It is generally, and usually, neither the fourier transform of the current correlator in the sample, nor its symmetrized version. However, the *excess* noise is instrumentation-independent, and is equal to the amplifier-gain squared times the difference in the fourier transform of the current correlator, in and out of equilibrium. This difference has a clear physical meaning: it is the difference in the power emitted *from* the sample and *into* the filter, in and out of equilibrium.

A. Cosine and sine components of time dependent functions

A real (and well behaved) function, $I(t)$, can be fourier-represented as:

$$I(t) = \frac{1}{2\pi} \int_0^\infty d\omega [I(\omega)e^{-i\omega t} + I^*(\omega)e^{i\omega t}]. \quad (1)$$

where $I(\omega) = \int_{-\infty}^\infty dt I(t)e^{i\omega t}$. If $I(\omega)$ is negligible outside a narrow bandwidth Δ_f around a center frequency $\Omega > 0$, $\Delta_f \ll \Omega$, then it is useful to write $I(t)$ in the form⁴:

$$I(t) = I_c(t) \cos \Omega t + I_s(t) \sin \Omega t, \quad (2)$$

where $I_c(t)$ and $I_s(t)$ are real and slowly varying - they have fourier-components only at frequencies smaller than Δ_f . $I_c(t)$ and $I_s(t)$ are called the *cosine* and *sine* components of $I(t)$. Defining the time average of $f(t)$ as:

$$\overline{f(t)} \equiv \frac{1}{T_0} \lim_{T_0 \rightarrow \infty} \int_t^{t+T_0} dt' f(t'), \quad (3)$$

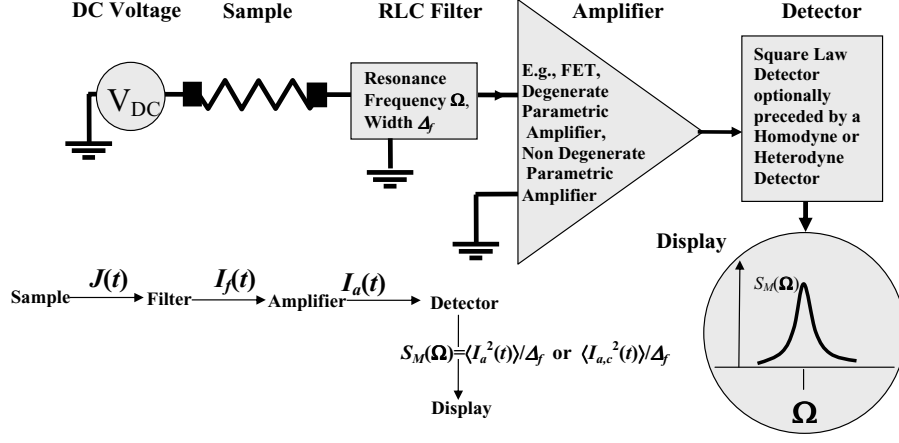


FIG. 1: Noise Measurement Procedure

then (if $\Delta_f \ll \Omega$) Eq. (2) implies: $\overline{I^2(t)} = \frac{1}{2}\overline{I_c^2(t)} + \frac{1}{2}\overline{I_s^2(t)}$. In a *stationary state*, the choice of the origin of time does not affect average quantities and therefore using and Eq. (2) for $t = 0$ and $t = \pi/(2\Omega)$ one gets:

$$\overline{I^2(t)} = \overline{I_c^2(t)} = \overline{I_s^2(t)}. \quad (4)$$

Moreover, in a stationary state time averaging may be replaced by averaging over realizations. We choose these realizations as repeated measurement on the same system at $N \gg 1$ different times, t_n , which are distributed over a whole time interval, T_0 , which is longer than other time scales in the systems: \hbar/eV , $\hbar/(k_B T)$, Ω^{-1} , and Δ_f^{-1} (in practice, usually the largest time is Δ_f^{-1} and therefore it is enough to require that the measurements are distributed over a time interval longer than it.). Thus, we can replace the time average: $\overline{f(t)} \rightarrow \langle f(t) \rangle = \frac{1}{N} \sum_{n=1}^N f(t_n)$.

B. Linear amplifiers

Below, we shall consider only *linear* amplifiers. An amplifier is linear if the current coming out of it, $I_a(t)$, is related to the one entering it, $I_f(t)$, by:

$$I_a(t) = G_1 I_{f,c}(t) \cos \Omega t + G_2 I_{f,s}(t) \sin \Omega t, \quad (5)$$

where $I_{f,c}(t)$ and $I_{f,s}(t)$ are the cosine and sine components of $I_f(t)$. and where at least one of the numbers G_1 and G_2 is large. If $G_1 = G_2$ the amplifier is called *phase-insensitive*. A phase insensitive amplifier does what one would naively expect from an amplifier - it just multiplies the incoming signal by a large number. If $G_1 \neq G_2$ the amplifier is called *phase sensitive* and it affects the two components of the incoming signal differently. An important special case is when $G_i \gg 1 \gg G_j$, $i \neq j$, where the amplifier amplifies only one of the components while disposing of the other.

Phase sensitive and insensitive amplifiers have similar classical behaviors, but may have different technical advantages. However, when quantum effects are important they differ fundamentally due to the limitations Heisenberg principle puts on them - see Ref. [4] and the Appendix.

In noise measurement in mesoscopic systems phase insensitive amplifiers are commonly used (e.g., in setups that include a field effect transistor) but we consider below also the phase sensitive case such as the degenerate parametric amplifiers⁴ because their quite developed technology (that was used, for example, in order to enable sensitive detection in experimental gravitational physics⁵) seems to be less familiar in the mesoscopic community and also in order to demonstrate the universality of our result.

II. NOISE AND EXCESS NOISE MEASUREMENT PROCEDURES

A. Noise measurement

In a typical noise measurement (shown in Fig. 1) at frequency Ω the current $J(t)$ that flows out of the sample (which is assumed to be in a stationary state, but not necessarily in equilibrium,) is filtered with an RLC circuit

around a resonance frequency $\Omega = (LC)^{-\frac{1}{2}}$. The current coming out of the filter, $I_f(t)$, is amplified, measured at some arbitrary time point, t , and the result of this measurement, $I_a(t)$, is squared (say, by a square law detector). This measurement is then repeated at $N \gg 1$ different times, $\{t_n\}$, and the results are averaged and divided by the filter bandwidth, Δ_f , giving a number which we shall denote by $S_M(\Omega)$,

$$S_M(\Omega) = \frac{1}{N\Delta_f} \sum_{n=1}^N I_a^2(t_n). \quad (6)$$

Finally, a voltage proportional to $S_M(\omega)$ is sent out to drive a display.

The explanation for why the above measurement is, at least in the classical case, a measurement of the current spectrum is given in the next section. Meanwhile we make the following comments:

1. The above procedure is not a measurement of a function of time. It is merely a series of *independent* samplings of a stationary process. Though many setups perform these samplings at times which are separated by a *constant* time interval, we stress that this is *not* essential (and actually may create confusion): the result will be the same even if the measurement is performed at *random times* as explained below Eq. (4)).

2. In some setups it is convenient to convert the signal to a low-frequency one and measure only the sine or cosine components, for example, by mixing the signal with a local oscillator (as is done in Heterodyne and Homodyne detection), that is, multiplying it by a pure sine or a cosine and averaging the result over time before squaring it. Such a mixing may introduce additional noise that should be taken into account. Numerical factors that may multiply the signal as a result of such a procedure are then cancelled by, e.g., calibration of the setup with respect to a source of noise with a known power spectrum such as a resistor in thermal equilibrium.

If the system, the filter and the amplifier are all in a stationary state, then according to Eq. (4) such a procedure yields the same $S_M(\Omega)$ as in the case of measuring and squaring the whole signal.

We shall always assume that the sample and the filter are in a stationary state, but we shall not necessarily assume that this is the case for the amplifier. It is typically the case for semiconductor amplifiers such as the field effect transistors used in noise measurement in mesoscopic systems but it is *not* the case in several types of parametric amplifiers⁴.

B. Excess noise measurement

In an excess noise measurement one subtracts the noise measured when the system is in equilibrium from that which is measured when the same system is driven out of equilibrium:

$$S_{M,excess}(\Omega) = S_{M,noneq}(\Omega) - S_{M,eq}(\Omega). \quad (7)$$

The excess noise is useful when one is interested in looking into the changes in the system which are due to driving it out of equilibrium. It is also useful when a particular setup (amplifier temperature and type, etc) affects the measurement by introducing an *additional* noise which is independent of the sample state, so by taking the difference between the two noise powers one can get rid of the instrumentation-dependent noise power.

Equilibrium properties, can not be described by the excess noise since by definition it vanishes in equilibrium.

Since in most cases mesoscopic samples are driven out of equilibrium by an external DC voltage, V , we shall consider the quantity:

$$S_{M,excess}(\Omega) = S_{M,V}(\Omega) - S_{M,0}(\Omega), \quad (8)$$

however, other means (e.g., by application of external radiation^{2,3}) can be used to drive the system out of equilibrium.

C. Statement of the problem

What quantity should one calculate in order to predict $S_{M,excess}(\Omega)$? Will this quantity depend on the properties of the sample only, or also on the particular experimental setup?

To answer these questions we first consider the classical case.

III. WHICH QUANTITY IS MEASURED IN A CLASSICAL NOISE MEASUREMENT?

A. The classical case without amplification

Consider a current $I(t)$ flowing in a system which is in a stationary state. Consider a long time interval $T_0 \gg \omega$ and define the *restricted fourier transform* of $I(t)$,

$$I_{T_0}(\omega) = \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} dt e^{i\omega t} I(t), \quad (9)$$

the *power spectrum* of $I(t)$,

$$S_I(\omega) = \lim_{T_0 \rightarrow \infty} \frac{|I_{T_0}(\omega)|^2}{T_0}. \quad (10)$$

and the *correlator* of $I(t)$,

$$c(\tau) = c(-\tau) = \overline{I(0)I(\tau)}. \quad (11)$$

The Wiener-Khinchin theorem⁶ states that

$$S(\omega) = S(-\omega) = \int_{-\infty}^{\infty} d\tau e^{-i\omega\tau} c(\tau), \quad (12)$$

$$c(\tau) = \frac{1}{\pi} \int_0^{\infty} d\omega \cos\omega\tau S_I(\omega). \quad (13)$$

and specifically also that,

$$c(0) = \overline{I^2(t)} = \frac{1}{\pi} \int_0^{\infty} d\omega S_I(\omega). \quad (14)$$

By its definition, a filter greatly reduces the fourier components at frequencies further than a band width Δ_f away from its center frequency, Ω . Therefore, assuming a regular behavior of $J_{T_0}(\omega)$, and a small Δ_f , the restricted transform of the current coming out of that filter, $I_{f,T_0}(\omega)$, is related to that of the incoming current, $J_{T_0}(\omega)$, by

$$\begin{aligned} I_{f,T_0}(\omega) &= \gamma J_{T_0}(\Omega) & |\omega - \Omega| &\lesssim \Delta_f, \\ I_{f,T_0}(\omega) &= 0 & |\omega - \Omega| &\gtrsim \Delta_f, \end{aligned} \quad (15)$$

where γ is constant. An example of a filter is shown in Fig. 2. For the moment we do not consider the possibility that the filter adds its own thermal noise to $I_f(t)$. Thus, according to their definitions, the power spectrum of $I_f(t)$ and $J(t)$ are related by:

$$\begin{aligned} S_f(\omega) &= \gamma^2 S_J(\Omega) & |\omega - \Omega| &\lesssim \Delta_f, \\ S_f(\omega) &= 0 & |\omega - \Omega| &\gtrsim \Delta_f. \end{aligned} \quad (16)$$

Applying Eq. (14) to $I_f(t)$, Eq. (12) to $J(t)$, and using Eq. (16) one gets

$$\frac{1}{\Delta_f} \overline{I_f^2(t)} = \gamma^2 S_J(\Omega) = \gamma^2 \int_{-\infty}^{\infty} d\tau e^{i\Omega\tau} \overline{J(0)J(\tau)}. \quad (17)$$

Eq. (17) clarifies what might be a confusing feature of a noise measurement with a filter: taking the square of the current coming out of the filter at one time yields information on the correlation in the current in the sample at two different times.

The average energy stored in an RLC filter is⁷:

$$\langle E_f \rangle = L \langle I_f^2 \rangle. \quad (18)$$

For later purposes we note that this equation is valid also in a stationary quantum state as a consequence of the virial theorem⁸. Since, for the moment, the amplification stage is ignored we have $I_f(t) = I_a(t)$. Therefore, making use of the definition of the measured noise, Eq. (6), we see that

$$S_M^{(\text{no ampl})}(\Omega) = \frac{1}{\Delta_f L} \langle E_f \rangle = \gamma^2 \int_{-\infty}^{\infty} d\tau e^{i\Omega\tau} \langle J(0)J(\tau) \rangle. \quad (19)$$

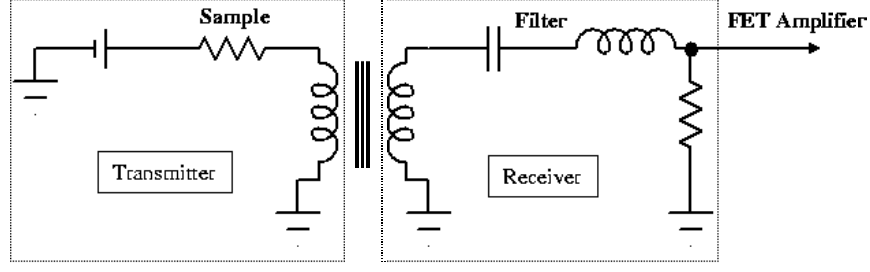


FIG. 2: Inductive coupling to an RLC filter

B. The classical case with amplification

We proceed now to include the amplification stage. In order to avoid complications due to the differences between different types of amplifiers we assume here that only the cosine component of the output current, $I_{a,c}(t)$, is measured, squared and averaged. Such an assumption enables us to assign a single gain, G , to the amplifier (whether it is phase sensitive or not) that multiplies the incoming signal. Thus, $I_f(t)$ enters the amplifier and $I_a(t)$ comes out of it. Then, the component $I_{a,c}(t) = GI_{f,c}(t)$ is measured and squared. But $\overline{I_{f,c}^2} = \overline{I_f^2}$ and therefore by Eqs. (17)-(19) one has:

$$S_M(\Omega) = \frac{G^2}{\Delta_f L} \langle E_f \rangle = \bar{G}^2 \int_{-\infty}^{\infty} d\tau e^{i\Omega\tau} \langle J(0)J(\tau) \rangle. \quad (20)$$

where $\bar{G} = \gamma G$.

C. Thermal noise produced by the setup

Even when considered classically, the measurement setup can perform according to Eq. (5) and (20) only as long as its components are operating at temperatures which are low compared with the signal power spectrum. At higher temperatures, one should take into account the thermal noise, $S_{N,T}(\omega)$, produced by these components and add it to the output signal. We shall not discuss the particular form of this noise, except for mentioning that in equilibrium and at low frequencies its contribution is $k_B T$ (the Nyquist-Johnson noise) times the amplifier-gain squared, and that it is independent of the input, i.e., of the state of the sample. Thus, we write:

$$S_M(\Omega) = \frac{G^2}{\Delta_f L} \langle E_f \rangle + S_{N,T}(\Omega) = G^2 \int_{-\infty}^{\infty} d\tau e^{i\Omega\tau} \langle J(0)J(\tau) \rangle + S_{N,T}(\Omega) \quad (21)$$

where $S_{N,T}(\Omega)$ is defined as the noise measured with no input.

Comments:

1. Other types of noise such (e.g., $1/f$ noise) may occur inside the setup components. However, unlike the noise required by thermodynamics (and in the quantum case also that required by the Heisenberg principle), these may in principle be eliminated and thus are not considered here.

2. Adding a setup noise which is assumed to be independent of the input, is justified by assuming that the statistical distributions of the state of the total system is a product of that of the amplifier (and the detector) and the filter+sample and that the coupling between those parts is weak. In the quantum case the assumption is that the density matrix of the system is a product of that of the amplifier (and the detector) and the sample+filter. For more details see the appendix.

D. Classical excess noise

According to Eqs. (8), (21), the excess noise in a classical measurement is:

$$S_{M,excess}(\Omega) = \frac{G^2}{\Delta_f L} \langle E_f \rangle_{excess} = \bar{G}^2 \int_{-\infty}^{\infty} d\tau e^{i\Omega\tau} \langle J(0)J(\tau) \rangle_{excess} \quad (22)$$

where

$$\langle E_f \rangle_{\text{excess}} = \langle E_f \rangle_V - \langle E_f \rangle_0, \quad (23)$$

and

$$\langle J(0)J(\tau) \rangle_{\text{excess}} = \langle J(0)J(\tau) \rangle_V - \langle J(0)J(\tau) \rangle_0, \quad (24)$$

are the differences in the filter energies and the correlators in and out of equilibrium. Eqs. (21) and (22) tell us what are the measured quantities in noise and excess noise measurements in classical situations, i.e., when quantum effects can be neglected. They show that although the measured noise, Eq. (21), is setup-dependent because the term $S_{N,T}(\Omega)$ depends on the setup type and temperature, the measured excess noise is not.

Eq. (22) also gives a simple physical picture to the excess noise: when a voltage is applied to the sample and drives it out of equilibrium, the current (or charge) fluctuations in the sample change (typically, they increase), and interact with the charges or currents in the RLC circuit (e.g., through capacitive or inductive coupling) causing an increase in the energy flow from the sample into the circuit in a similar way to that in which the current fluctuations in an antenna of a classical transmitter emit energy into a receiver (Fig. 2). Part of this energy is accumulated in the capacitor and the inductor and part is dissipated in the resistor. Eventually the system arrives at a stationary (though not an equilibrium) state where the filter energy is higher than before. The measured excess noise is simply this increase in the filter energy multiplied by the amplifier gain.

Having obtained a detailed picture of the classical noise measurement we are now ready to analyze the quantum case.

IV. WHICH QUANTITY IS MEASURED IN QUANTUM NOISE MEASUREMENT?

When the measured frequency is higher than the temperature of the sample or the setup, quantum effects become important. One may then consider replacing the current $J(t)$, and the average over realizations $\langle J(0)J(\tau) \rangle$, in Eqs. (21) and (22) by, respectively, the Heisenberg current operator of the electrons in the sample, $\hat{J}(t)$, and the expectation value of the product of operators, $\hat{J}(0)\hat{J}(\tau)$, in the quantum stationary state of the system. However, in attempting to do so one immediately encounters the following two questions (which are answered below):

1. The current operator does not commute with itself at different times (the product $\hat{J}(0)\hat{J}(\tau)$ is not Hermitian) and therefore it is not clear in which order, $\hat{J}(0)\hat{J}(\tau)$ or $\hat{J}(\tau)\hat{J}(0)$ the product should be written or whether it should be replaced by its *symmetrized* version $(\hat{J}(0)\hat{J}(\tau) + \hat{J}(\tau)\hat{J}(0))/2$, (which is Hermitian) as is customarily suggested in text books⁹.
2. What are the properties of the setup noise (the analog $S_{N,T}(\Omega)$) in the quantum case, and specifically, does it vanish in the limit of zero temperature as was the case in the classical regime?

A. The quantum case without amplification

Consider first a mesoscopic system, e.g., a ballistic quantum point contact between two ohmic contacts, in which a DC voltage is applied to the left contact and the current flowing out of the right one interacts with the current in an RLC circuit (modelled by an harmonic oscillator with a small damping) through an inductive coupling of the form (see Fig. 2):

$$\alpha \hat{J}(t) \hat{I}_f(t).$$

The above system was considered in Refs.¹⁰ and¹¹ (in the limit of small α and Δ_f). It was shown, that as a result of this interaction there is an energy flow between the electronic system and the filter. The current fluctuations in the electronic system excite the harmonic modes of the filter in a similar way to that in which current fluctuations in an antenna excite the photon modes in the electromagnetic field of the vacuum. As a result of switching the *interaction* on adiabatically while keeping the DC voltage constant, (unlike what is considered above where the *voltage* is switched on), the energy of the filter was found to increase by an amount of of^{10-11} :

$$\begin{aligned} \delta \langle E_f \rangle &= L(\langle \hat{I}_f^2(t) \rangle_\alpha - \langle \hat{I}_f^2(t) \rangle_{\alpha=0}) = \\ &\gamma^2 [(N+1)S_J(\Omega) - NS_J(-\Omega)] = \gamma^2 [S_J(\Omega) - 2N\hbar\Omega G_d(\Omega)], \end{aligned} \quad (25)$$

where $S_J(\Omega)$ is the transform of the *nonsymmetrized* quantum correlator,

$$S_J(\Omega) = \int_{-\infty}^{\infty} d\tau e^{i\Omega\tau} \langle \hat{J}(0) \hat{J}(\tau) \rangle, \quad (26)$$

where N is the average number of quanta in the oscillator, and $G_d(\Omega)$ is the differential conductance¹². γ^2 is equal to α^2 times some multiplicative factors (which anyhow cancel in the setup calibration). $S_J(\Omega)$ has a physical meaning: it is proportional to the fermi-golden rule emission-rate of quanta of $\hbar\Omega$ from the sample into the filter¹¹. Similarly, $S_J(-\Omega)$ is proportional to the absorption rate. Thus, Eq. (25) has a simple physical meaning: the change in the filter energy correspond to the spontaneous emission from the sample plus the net energy flow due to the difference between the induced emission and absorption.

In order to obtain the change in the filter energy due to the application of the voltage, one has to calculate the difference between finite and zero voltage: $\langle E_f \rangle_{\text{excess}} = \delta \langle E_f \rangle_V - \delta \langle E_f \rangle_0$. One obtains:

$$\langle E_f \rangle_{\text{excess}} = \gamma^2 \int_{-\infty}^{\infty} d\tau e^{i\Omega\tau} \langle \hat{J}(0) \hat{J}(\tau) \rangle_{\text{excess}}, \quad (27)$$

where it was assumed that the voltage-dependence of the differential conductance is weak. Eq. (27) expresses the change in the filter energy when the sample is in and out of equilibrium. We are now ready to proceed to the final step and take the amplification into account.

B. The quantum case with amplification

Consider the current $\hat{I}_f(t)$ in an RLC circuit which serves as the input signal of a linear amplifier. The quantum theory of linear amplifiers⁴ specifies what limitations the Heisenberg principle puts on their performances. The limitations relevant to our case can be summarized as follows:

An amplifier that amplifies both sine and cosine components of the input signal *must* add noise, which we will denote by $S_{N,Q}(\Omega)$, to the measured signal, and this noise does not vanish at zero temperature. Therefore, a phase insensitive amplifier must add noise to the measured signal. However, the added noise is not necessarily distributed evenly between the two components. The amount of noise which is added, say, to the cosine component, depends on the particular amplification setup and temperature. The fluctuation of the current coming out of a phase insensitive amplifier are given by the form⁴:

$$S_M(\Omega) = \Delta_f^{-1} \langle \hat{I}_a^2(t) \rangle = G^2 \Delta_f^{-1} \langle \hat{I}_f^2(t) \rangle + S_{N,Q}(\Omega) \quad (28)$$

where $S_{N,Q}(\Omega)$ depends only on the properties and state of the amplifier and detector while $\langle \hat{I}_f^2(t) \rangle$ depends only on those of the filter. Similarly, the fluctuation of the current coming out of a phase sensitive amplifier (which can be, ideally, noiseless and which amplifies only the cosine component and disposes of the sine component) are given by the form:

$$S_M(\Omega) = \Delta_f^{-1} \langle \hat{I}_{a,c}^2(t) \rangle = G^2 \Delta_f^{-1} \langle \hat{I}_{f,c}^2(t) \rangle. \quad (29)$$

A brief review on the origin of the additive form of the right hand side of Eq. (28) is given in the appendix.

V. EXCESS QUANTUM NOISE MEASUREMENT

Since the filter is in a stationary state, one has $\langle \hat{I}_f^2 \rangle = \langle \hat{I}_{f,c}^2 \rangle = \frac{1}{L} \langle E_f \rangle$ and therefore, Eqs. (18), (28) and (29) yield:

$$S_{M,\text{excess}}(\Omega) = G^2 (L \Delta_f)^{-1} \langle E_f \rangle_{\text{excess}}. \quad (30)$$

Eqs. (27) and (30) imply

$$S_{M,\text{excess}}(\Omega) = \bar{G}^2 \int_{-\infty}^{\infty} d\tau e^{i\Omega\tau} \langle \hat{J}(0) \hat{J}(\tau) \rangle_{\text{excess}}, \quad (31)$$

where $\bar{G} = \gamma G$.

Eq. (31) is our main result. It shows that in order to predict the result of an excess noise measurement one should calculate the correlators (no symmetrization is required) in and out of equilibrium and take the difference between them. It also shows that there will be no contribution from the zero-point fluctuations since $S(\Omega > 0)$ (the *emission* spectrum) does not contain such contribution - the zero point fluctuations can not emit energy.

When the *sample* (but not necessarily the setup) is at zero temperature the correlator vanishes in equilibrium since the sample can not emit anything. Therefore:

$$S_{M,excess}(\Omega) = G^2 \int_{-\infty}^{\infty} d\tau e^{i\Omega\tau} \langle \hat{J}(0) \hat{J}(\tau) \rangle, \quad k_B T_s = 0. \quad (32)$$

We note that the equality in Eq. (31) is not term-by-term. The excess noise is equal to the excess of the correlator but the noise by itself is not equal to the correlator by itself since according to Eq. (28) the former depends also on the setup properties while the latter depends only on the sample properties.

The derivation of Eq. (31) is valid for all linear amplifiers used in noise measurements in mesoscopic systems and the parametric ones which are analyzed in Ref. [4]. The condition that the conductance G_d remains constant when the DC voltage is turned on, that was used in deriving this equation, may be understood by considering the zero temperature case: the excess noise is the power flow of energy from the sample into the filter. In order to enable an efficient measurement of this power an impedance matching is needed between the sample, the transmission lines and the filter. If the sample conductance is very different in its equilibrium and nonequilibrium states, then, initially good impedance-matching with the detector that enables an efficient power flow in equilibrium, means a bad impedance matching out of equilibrium with an inefficient power flow. In such a case one should correct Eq. (31) by taking into account the different impedance ratios in and out of equilibrium.

VI. SUGGESTED VERIFICATIONS OF THE THEORY

A straightforward way to verify Eq. (32) is to measure the excess noise in a single-channel ballistic quantum point contact at high frequencies, $\hbar\Omega \gtrsim eV$. For such a system the nonsymmetrized correlator is given by (see Ref. [14] for the symmetrized version, and [15] and [13] for the nonsymmetrized one):

$$S(\Omega, T_s, V) = \frac{e^2}{h} |t|^2 (1 - |t|^2) \sum_{\epsilon=\pm 1} F(\hbar\Omega + \epsilon eV) + \frac{e^2}{h} |t|^4 F(\hbar\Omega) \quad (33)$$

where $F(x) = x(e^{x/k_B T_s} - 1)^{-1}$, T_s is the sample temperature and $|t|^2$ is the transmission of the channel. According to our theory, such an excess noise measurement would yield $S(\Omega, T_s, V) - S(\Omega, T_s, 0)$ for any amplifier type or temperature, while without taking the excess the result will generally depend substantially on type of the setup and its temperature. In particular, for $T_s \ll eV, \hbar\Omega$, the excess noise measurement will yield $S(\Omega, 0, V)$, i.e., the nonsymmetrized correlator (with no contribution from the zero point fluctuations) while without taking the excess the result will generally differ from both the non-symmetrized and the symmetrized correlators and will depend on the particular setup.

VII. APPENDIX. NOISE ADDED IN AMPLIFICATION

A. Requirements from quantum linear amplifier output

Consider an RLC circuit (see e.g., Fig. 2), which we shall call 'the input', connected into a linear amplifier. Let $\hat{I}_f(t)$ be the Heisenberg operator of the current in the input. This operator acts on the degrees of freedom of the circuit and therefore its expectation values are determined when the circuit state is given. Let $\hat{I}_a(t)$ be the Heisenberg operator of the current at the output port of the amplifier. In general, this operator acts on both the input and the amplifier degrees of freedom. For an ideal linear amplifier:

$$\hat{I}_a(t) = \hat{I}_{a,c}(t) \cos \Omega t + \hat{I}_{a,s}(t) \sin \Omega t = G_1 \hat{I}_{f,c}(t) \cos \Omega t + G_2 \hat{I}_{f,s}(t) \sin \Omega t. \quad (34)$$

Note that in such an ideal case the output operator, $\hat{I}_a(t)$, acts only on the degrees of freedom of the input (and not any of the setup) - the input state determines completely the output independently of the amplifier state. However, the Heisenberg principle limits on the possibility of realizing such a device: it requires the sine and the cosine components

to obey a minimum uncertainty relation (see Eq. (2.19) in Ref. [4]). Suppose one constructs a minimum-uncertainty state for $\hat{I}_{a,c}(t)$ and $\hat{I}_{a,s}(t)$, so that they can be measured simultaneously at the maximum accuracy permitted without violating the Heisenberg principle. If $G_1 G_2 > 1$ (as is the case for a phase insensitive amplifier where $G_1 = G_2 \gg 1$) then $\hat{I}_{f,c}(t)$ and $\hat{I}_{f,s}(t)$ could have been measured simultaneously up to an accuracy which is $G_1 G_2$ times better than allowed simply by measuring $\hat{I}_{a,c}(t)$ and $\hat{I}_{a,s}(t)$, and then applying Eq. (34). Therefore, an amplifier with $G_1 G_2 > 1$ and specifically a phase insensitive amplifier is forbidden in quantum mechanics.

One way to overcome these limitations is to build an amplifier in which, for example, $G_1 \gg 1$, $G_1 G_2 = 1$, that is, a phase sensitive amplifier that amplifies the cosine component and diminishes the sine component so that in the limit of $G_1 = G \rightarrow \infty$ we can write:

$$\hat{I}_a(t) = G \hat{I}_{f,c}(t) \cos \Omega t \quad (35)$$

An example for such a device is the degenerate parametric amplifier⁴. Another way to overcome the above limitations is to allow $G_1 G_2 > 1$, (and in particular $G_1 = G_2 = G \gg 1$), but add to the right hand side of Eq. (34) an additional term that will operates on the amplifier degrees of freedom so that:

$$\hat{I}_a(t) = G \hat{I}_f(t) + \hat{I}_{N,Q}(t), \quad (36)$$

where $\hat{I}_{N,Q}(t)$ is an operator acting on the amplifier degrees of freedom such as the electronic state in a field effect transistor or the idler resistor state in a non-degenerate parametric amplifier.

B. Independence of the amplifier noise of the sample state

Let us assume that the input is in a stationary state in which the average currents vanish and take the expectation square of the output current, as is done in a noise measurement. In order to do so we should specify the state of the system or more generally, the density matrix. Here we make an important assumption that the total density matrix of the system, ρ_s is a product of the amplifier density matrix, ρ_a , and the input one, ρ_f : ρ_a and ρ_f , $\rho_s = \rho_f \rho_a$. Such an assumption is justified, e.g., when the interaction between the amplifier and the input is small compared to their coupling with the thermal baths that determine their temperatures. Then using Eq. (35) and averaging over time much larger than Ω^{-1} one has

$$\langle \hat{I}_a^2(t) \rangle_s = \frac{1}{2} G^2 \langle \hat{I}_{f,c}^2(t) \rangle_{f,a} = \frac{1}{2} G^2 \langle \hat{I}_{f,c}^2(t) \rangle_f, \quad (37)$$

where $\langle A \rangle_x = \text{Tr} \rho_x A$. This relationship is identical to Eq. (29) except for the factor 1/2 which appeared due to different definitions of the gain. In the phase insensitive case one gets from Eq. (36)

$$\begin{aligned} \langle \hat{I}_a^2(t) \rangle_s &= G^2 \left[\langle \hat{I}_f^2(t) \rangle_f + \langle \hat{I}_f^2(t) \rangle_a + 2 \langle \hat{I}_f(t) \rangle_f \langle \hat{I}_a(t) \rangle_a \right] \\ &= G^2 \langle \hat{I}_f^2(t) \rangle_f + S_{N,Q}(\Omega) \Delta_f \end{aligned} \quad (38)$$

where $S_{N,Q}(\Omega) \Delta_f = G^2 \langle \hat{I}_f^2(t) \rangle_a$, as in Eq. (28). Eqs. (37) and (38) shows that the amplifier noise is *additive* to the input noise (in Eq. (37) it is trivially zero), at least as long as one assumes that the density matrix of the system can be factorized as described above.

VIII. ACKNOWLEDGMENT

This project was supported by the Israel Science Foundation, by the German-Israeli Foundation (GIF), Jerusalem, and by the Maurice and Gabriella Goldschleger Center for Nanophysics at the Weizmann Institute. E. Comforti, R. dePiccioto, C. Glattli, M. Heiblum, W. Oliver, D. Prober, M. Reznikov, U. Sivan and A. Yacoby are warmly thanked for many discussions, and for providing detailed descriptions of their experimental setups. U. G. would like to express special thanks to B. Doucot for his support and for many very helpful discussions.

¹ M. Reznikov, M. Heiblum, H. Shtrikman, and D. Mahalu, Phys. Rev. Lett. **75**, 3340 (1995); A. Kumar, L. Saminadayar, D. C. Glattli, Y. Jin and B. Etienne, Phys. Rev. Lett. **76**, 2778 (1996); R. dePiccioto et al. Nature, **389**, 162 (1997); L. Saminadayar et al. Phys. Rev. Lett. **79**, 2526 (1997).

- ² R.J. Schoelkopf, A.A. Kozhevnikov, D.E. Prober, M.J. Rooks, Phys. Rev. Lett. 80, 2437 (1998)
- ³ L.-H. Reydellet, P. Roche, D. C. Glatthli, B. Etienne and Y. Jin, cond-mat/0206514 v2 3 Jul 2002.
- ⁴ B. Yurke and J. S. Denker, Phys. Rev. A. **29**, 1419 (1984).
- ⁵ See, e.g., K. S. Thorne et al. in *Sources of Gravitational Radiation*, L. Smarr (ed.), Cambridge University Press, London, p. 49 (1979).
- ⁶ E.g., see C. W. Gardiner, *Handbook of Stochastic Methods*, 2nd ed., Sec. 1.4.2, p. 15, Springer-Verlag, (1990)
- ⁷ See, e.g., E. M. Purcell, *Electricity and Magnetism*, Sec. 8.5, McGraw-Hill (1965).
- ⁸ E.g., see under 'Virial Theorem' in C. Cohen-Tannoudji, B. Diu and F. Laloe, *Quantum Mechanics*, 2nd ed., ch. 3, p. 344 (1977).
- ⁹ See e.g., L. D. Landau and E. M. Lifshitz, *Statistical Physics Part 1*, 3rd ed., Sec. 118, p. 360, Eq. 118.4, Butterworth Heinemann (1997).
- ¹⁰ G. Lesovik and R. Loosen, JETP Lett., **65**, 295 (1997).
- ¹¹ U. Gavish, Y. Levinson and Y. Imry, Phys. Rev. **B 62** R10637 (2000). Y. Levinson and Y. Imry, Proc. of Nato ASI "Mesoscope 99" in Ankara/Antalya, Kluwer 1999.
- ¹² In deriving the right equality in Eq. (25) the fluctuation-dissipation theorem (or generalized Kubo formula) was used *out of equilibrium*. The theorem provides a relation between the negative and positive frequency branches of the correlator. See Eq. (21) in Sec. 4 of Ref. [13].
- ¹³ U. Gavish, Y. Levinson and Y. Imry, Quantum noise, Detailed Balance and Kubo Formula in Nonequilibrium Systems, in *Electronic Correlations: from Meso- to Nano-Physics*, Proc. of the XXXVI Rencontres de Moriond, March 2001, T. Martin, G. Montambaux and J. T. T. Vàn, eds., EDP Sciences (2001), p. 243.
- ¹⁴ S.-R. Eric Yang, solid state communications, **81**, 375, (1992).
- ¹⁵ R. Aguado and L. P. Kouwenhoven, Phys. Rev. Lett., **84**, 1986 (2000)